MID-SEMESTER EXAMINATION
M. MATH II YEAR, I SEMESTER 2016-2017
FOURIER ANALYSIS

Max. Score:100 Time limit: 3hrs.

1. Find the Lebesgue points of the function
$f(x) = \begin{cases} 0 \text{ if } x < 0 \\ 1 \text{ if } x \geq 0 \end{cases}$ [15]

2. Consider the set $M = \{ \sum_{n=-N}^{N} c_j e^{ijx} : c_1, c_2, \ldots, c_N \in \mathbb{C}\}$ where N is a given positive integer. Is this a closed subspace of $L^1(\mu)$ (where μ is the normalized Lebesgue measure on $[0, 2\pi]$)? Justify. [15]

3. Prove that $x = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{(2n-1)^2}$ for $0 \leq x \leq \pi$. [15]

4. Let $f \in L^1(\mu)$ and $S_N(x) = \sum_{n=-N}^{N} \hat{f}(n) e^{inx}$. Show that $\lim_{N \to \infty} \frac{S_N(x)}{N}$ exists for every x and find the limit. [15]

5. If $f \in L^1(\mu)$ (μ as in Problem 4) and if f is continuous at 0 show that $\sum_{n=-N}^{N} (1 - \frac{|n|}{N+1}) \hat{f}(n) \to f(0)$ as $N \to \infty$. [20]

6. If $\sum_{n=1}^{\infty} |a_n \cos(nx) + b_n \sin(nx)| < \infty$ for all x in a set of positive measure show that $\sum_{n=1}^{\infty} |a_n| < \infty$ and $\sum_{n=1}^{\infty} |b_n| < \infty$. [20]

Hint: proceed as in Cantor -Lebesgue Theorem and use the fact that $\int_E |\cos(nx - \theta_n)|\, dx \geq \int_E \cos^2(nx - \theta_n) dx$.